Magnetic field production during preheating at the electroweak scale

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We study the generation of magnetic fields during preheating within an scenario of hybrid inflation at the electroweak (EW) scale. We find that the non-perturbative and strongly out-of-equilibrium process of magnetic field production occurs along the lines predicted by Vachaspati many years ago. The system starts in the false vacuum at the end of inflation, and very quickly the initial quantum fluctuations of the Higgs field get amplified via long wavelength spinodal instabilities. The subsequent nucleation of the random Gaussian Higgs field bubbles (lumps) leads to EW symmetry breaking, and to the creation of Z-strings, which soon decay, along with longwave magnetic flux tubes with nontrivial helicity. The intensity and scales in these helical magnetic fields are consistent with their later development into the microgauss fields observed in galaxies and clusters of galaxies.

The origin of primordial magnetic fields (PMF) is one of the remaining puzzles in cosmology [1]. Magnetic fields play an important role in the evolution of the primordial plasma in the early universe, and in the propagation of high energy cosmic rays in our galaxy and beyond. They may influence galaxy and large scale structure formation, and may generate a stochastic background of gravitational waves.

Magnetic fields have been measured on the scales of galaxies and clusters of galaxies with a magnitude of order microgauss [2]. There is even some evidence of their presence on scales of superclusters, and associated with quasars at redshifts $z \sim 2$. The main difficulty in understanding their origin is not in their magnitude but in its correlation scale. The μG order of magnitude of galactic MF could be explained by an amplification via a dynamo mechanism initiated by a tiny seed, $B \sim 10^{-30}$ G, in the context of gravitational collapse in a flat ΛCDM model. Their magnitude on clusters scales is much more difficult to explain via a dynamo mechanism [1]. In any case, an initial seed will be required. There are both astrophysical and cosmological theoretical models trying to account for the origin of the primordial seed: Biermann battery in intergalactic shocks, stellar magnetic winds (like in our Sun), supernova explosions, galactic outflows in the intergalactic medium (IGM), quasar outflows of magnetized plasma into the intra-cluster medium (ICM) [1]; as well as from early universe phase transitions [3], from magnetic helicity at the EW transition, together with the baryon asymmetry of the universe (BAU) [4], via hypercharge and hypermagnetic field generation before the EW transition [5], from second order cosmological perturbations from inflation [6], from reheating after inflation [7], or produced during inflation [8], etc.

The ubiquity of microgauss MF on all scales suggests a cosmic origin. Could they be generated in the early universe and then redshifted until today? Magnetic fields could have appeared as the electroweak force broke into weak interactions plus electromagnetism, at a scale $T_{\rm EW} \sim 100$ GeV, when the universe had an energy density $\rho_{\rm EW} \sim 10^8$ GeV^4. The universe was (or became) radiation dominated at that time, and the energy density carried by magnetic fields could correspond to a fraction f of the total. If we redshift this energy density until today ($T_0 = 2.725$ K) we get $\rho_{\rm B}(t_0) = (T_0/T_{\rm EW})^4 \, f \, \rho_{\rm EW} \sim 3 \times 10^{-53} \, f \, {\rm GeV}^4 = 0.4 \, f \, {\rm eV/cm}^3 = (5 \, f^{1/2} \, \mu{\rm G})^2/(8\pi)$, where we have used 1 G $= 1.95 \times 10^{-20}$ GeV^2. This would be enough to explain the cluster and supercluster values, and would perhaps require a mild dynamo mechanism to amplify it to galactic values (if the fraction $f \ll 1$).

However, a priori, it is not so clear how one can obtain the large correlation length of magnetic fields observed at galactic and cluster scales. Any physical mechanism that creates magnetic fields must necessarily be causal, but at high temperatures in the early universe there is also a natural coherence scale given by the physical horizon. At the EW scale the particle horizon is 10^{-10} lightseconds (~ 3 cm), which today corresponds to a comoving scale of ~ 1 AU, clearly insufficient when compared even with the irregular (turbulent) component of the galactic magnetic field ($L \sim 100$ parsecs), not to mention the regular (uniform) component, which has correlations $L \sim 10$ kpc. It thus seems apparently impossible to explain with our mechanism the coherent magnetic fields observed on galaxy clusters and supercluster scales (of order 10 Mpc) with intensities of order μG to nG. Nevertheless, if we assume that the plasma after the EW transition is sufficiently turbulent to maintain magnetic fields of the largest possible coherence scales via inverse cascade, then we could reach cosmological scales today. The largest coherence scale is the physical horizon. If a strong inverse cascade is active, then the coherence length of the magnetic fields could grow as fast as the horizon, i.e. like the scale factor *squared* during the radiation dominated era. This optimal situation is only attainable in the presence of a plasma, and thus is bound to stop acting at photon decoupling, when the universe becomes neutral. Since then, the correlation length can only grow with the expansion of the universe, as the scale factor. Taking these facts into account, and using the adiabatic expansion relation, we can compute the coherence scale of the field in terms of that at the electroweak scale, $\xi_0 = \xi_{\rm EW} \left(\frac{a_{\rm dec}}{a_{\rm EW}} \right)^2 \frac{a_0}{a_{\rm dec}} = 3 \, {\rm cm} \left(\frac{T_{\rm EW}}{T_{\rm eq}} \right)^2 \frac{T_{\rm eq}}{T_0} \sim 6 \times 10^{25} \, {\rm cm} = 20$ Mpc, where we have

made the approximation that matter-radiation equality and photon decoupling occurred more or less simultaneously (a careful computation gives only a minor correction). The surprising thing is that this simple calculation gives precisely the order of magnitude for the largest correlation length of cosmic MF ever observed (i.e. cluster scales). If the μG magnitude of the primordial MF seed, arising from simple scaling down the energy density since the EW transition, seemed a curious coincidence, the fact that an inverse cascade could also be responsible for the observed correlation length becomes a suspicious coincidence. These observations have triggered a large number of investigations devoted to the analysis of magnetogenesis during the electroweak phase transition [3, 4, 5].

In this Letter we address the question of whether the required conditions –a significant fraction of energy density in magnetic fields with a large helical component and sustained periods of inverse cascade – can be obtained within a simple and concrete scenario of low-scale hybrid inflation [9]. We analyze whether large scale magnetic fields are generated during the preheating and reheating periods which lead to EW symmetry breaking. Previous attempts to analyze magnetogenesis during reheating as a consequence of spinodal instability can be found in Ref. [7]. Our approach differs significantly and is based on the use of the classical approximation: integration of the non-linear equations of motion with stochastic initial conditions determined by matching the quantum expectation values at the end of inflation. The numerical implementation is based upon discretization of our system onto a space-time lattice. Our results indicate that a mechanism resembling that introduced by Vachaspati [4] could have taken place in the early universe. The mechanism relies on the fact that an inhomogeneous Higgs background may act as a source for magnetic fields. The effect would come from the existence of non-trivial gradients of the Higgs field phase which would generate W-currents and magnetic fields during a cold EW transition [9]. These gradients are usually associated with the appearance of unstable Z-strings, together with the helical magnetic field necessary for growth into large scales via MHD turbulence [4]. As we will see, our results show that during the initial stage of electroweak symmetry breaking (EWSB), our setup is indeed characterized by the appearance of Z-strings and magnetic strings with non-trivial helicity.

We consider a specific model of Hybrid inflation with the field content of the gauge plus scalar sector of the Standard Model coupled to a singlet scalar field, the inflaton [9]. The gauge sector includes both the SU(2) and the hypercharge U(1) fields with corresponding gauge couplings $g_{\rm W}$ and $g_{\rm Y}$. The scalar sector contains the 4-real component Higgs field $\phi_\mu(x)$, and a singlet inflaton field $\chi(x)$ coupled to the Higgs via the term $g^2 \phi_\mu^2 \chi^2$. Our initial condition assumes that at the end of inflation both the Higgs and the gauge fields are in the de Sitter vacuum. In fact, since for EW-scale inflation the rate of expansion is negligible compared to any other
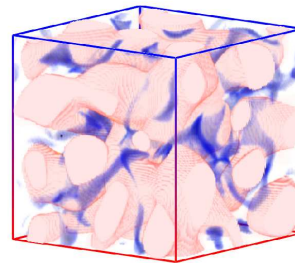

FIG. 1: The location of the Higgs lumps (light/red) and the magnetic field flux tubes (dark/blue) at $mt = 15$.

scale, $H \approx 10^{-5}$ eV $\ll v = 246$ GeV, the de Sitter vacuum state is indistinguishable from the Minkowski vacuum for the range of momenta we are considering. The inflaton field is given by the homogeneous mode alone, which is decreasing with time. This time dependence translates, through the Higgs-inflaton coupling, into a time dependence of the Higgs mass-squared, which changes from positive to negative inducing an exponential growth of the Higgs low momentum modes in a process known as "tachyonic preheating" [10]. The remaining non-tachyonic fields are populated through their interaction to the Higgs. Fermionic fields can be safely ignored from the analysis since the perturbative decay of the Higgs into quarks and leptons occurs within a time scale $mt \sim 10^3 - 10^4$, many times bigger than the scales we are considering in our simulations.

There have been several numerical studies of this model [10, 11], in an attempt to explain the BAU in this context [9, 12, 13], as well as the production of a gravitational wave background [15]. In this paper, the Hypercharge gauge field is included for the first time(preliminary results were discussed in Ref.[14]). This will allow us to study the generation of U(1) electromagnetic fields during preheating. In order to analyze the results we first have to provide a definition of the U(1)$_{\rm em}$ content of the SU(2)×U(1) fields in the Lagrangian. We will be using a gauge invariant definition of Z vector potential on the lattice as

$$\hat{Z}_\mu(n) = \frac{\text{Tr}\Big[-i\hat{n}\hat{D}_\mu(\hat{\Phi}(n))\hat{\Phi}^\dagger(n)\Big]}{2|\hat{\phi}(n)||\hat{\phi}(n+\mu)|}, \qquad (1)$$

where $\hat{\phi}_\mu(n)$ are the lattice Higgs field components, with modulus $|\hat{\phi}(n)|$. In matrix notation, $\hat{\Phi}(n) = \phi_0\mathbf{I} + i\phi_a\tau_a$. We have introduced the adjoint unit vector $\hat{n} = n_a\tau_a$, with components $n_a(n) = \varphi^\dagger(n)\tau_a\varphi(n)/|\varphi(n)|^2$, with $\varphi(n) = \Phi(n)(1,0)^{\rm T}$ the Higgs doublet, and $\hat{D}_\mu$ is the SU(2)×U(1) lattice covariant derivative. Note that our definition of the Z vector potential corresponds to the standard one in the unitary gauge. We can then compute the field strength of the U(1)$_{\rm em}$ field in terms of the Z and Hypercharge field strengths:

$$\hat{F}_{\mu\nu}^\gamma(n) = \sin^2(\theta_W)\hat{F}_{\mu\nu}^{\rm Z}(n) - \hat{F}_{\mu\nu}^{\rm Y}(n). \qquad (2)$$
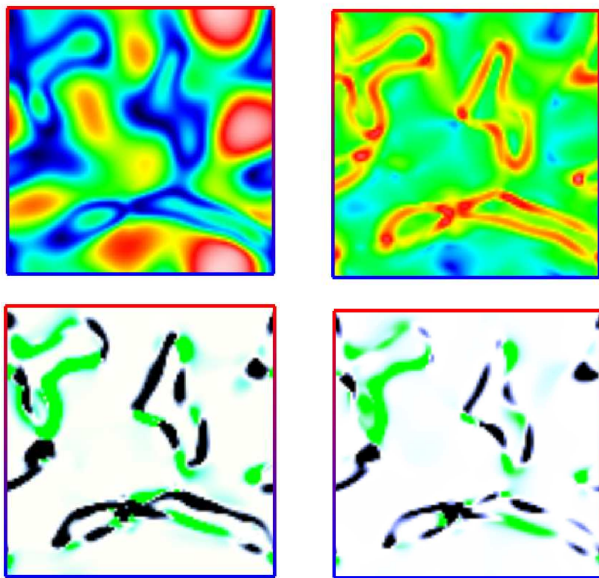
FIG. 2: 2D contour plots of the Higgs norm (top left), magnetic field flux tubes (top right), Z-string helicity (bottom left) and magnetic field helicity (bottom right) at $mt = 15$. Note the precise spatial correlation.

This procedure preserves the $U(1)_{em}$ gauge-invariance on the lattice.

As mentioned previously, there are two essential ingredients in Vachaspati's proposal for magnetogenesis: the existence of inhomogeneities in the Higgs field and the generation of a non-trivial magnetic field helicity. Both of them are present in the first stages of evolution right after inflation ends. This might seem surprising since initially both hypercharge and $SU(2)$ magnetic fields are zero. The way it comes about is by noticing that local variations of the orientation of the Higgs field induce via Eq. (2) a non-vanishing contribution to the electromagnetic field. The initial conditions for the classical evolution are given by a Higgs field as a Gaussian random field. This initial distribution carries essential information about how the process of EW symmetry breaking takes place. It is, for instance, the seed for the growth of bubble shells which arise from lumps in the initial distribution of the Higgs field norm. We have observed that it also determines the way in which magnetic fields are seeded. At later times, after EWSB has occured, the correlation between the Higgs field norm and magnetic fields is manifest. In Fig. 1 we show a snapshot of both quantities at a time ($mt = 15$) after EWSB has occurred and bubble shells (in red) fill almost all the volume of the box. Magnetic fields (in blue) are localized in string-like structures concentrated in the region between bubbles. It is there where the Higgs field remains closer to the false vacuum for a longer period of time giving rise to larger gradients in the Higgs field phases. This linkage between magnetic strings and Higgs field minima is even more evident in the two dimensional contour plots presented in Fig. 2. As mentioned previously part of this structure
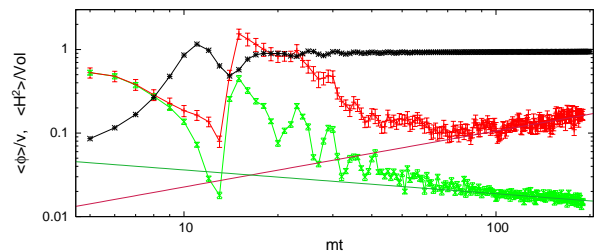

FIG. 3: Time evolution of the Higgs v.e.v. (black) and the square of the helicity in Z (green) and $U(1)$ (red) magnetic fields normalized to the physical volume $\mathcal{V}$. The Z-helicity has been rescaled by $\tan^2 \theta_W$. For $m_h = 2m_w$, the corresponding slopes are: Photon field: 0.68(7) and Z field: $-0.27(4)$.

was already present in our initial condition ($mt = 5$), in the absence of $SU(2)$ and hypercharge magnetic fields, but it has become far more complex at this stage.

It is interesting to point out that previous analysis of the cold EW transition, neglecting the effect of hypercharge, have revealed that regions between bubbles are also sources of Chern-Simons number through the appearance of sphaleron-like configurations attached to the location of zeroes in the Higgs field norm [12, 13]. Since for non-zero Weinberg angle sphalerons are like magnetic dipoles, it is tempting to correlate the observed $U(1)$ magnetic strings to the alignment of sphaleron dipoles. A detailed investigation of this issue is beyond the scope of this paper. In any case, the connection between Chern-Simons number and magnetic structures is more explicit when analyzing the generation of magnetic helicity, which can be regarded as the Abelian counterpart of the Chern-Simons number. During these first stages of evolution the magnetic and Z-boson helicities are indeed tracking the magnetic strings (a clear example is shown in Fig. 2, comparing top and bottom boxes). The proportionality of the Z-boson and photon helicities in the initial condition is a direct consequence of our definition of the fields. Nonetheless this correlation is still preserved once gauge fields and non-linearities have started to play a role.

The previous figures correspond to a snapshot at $mt = 15$, when the string structure comes out to be stronger. It is important to ascertain whether the later stages of evolution preserve relic magnetic fields with non trivial magnetic helicity. This quantity is defined as:

$$H = \int d^3 x \vec{A} \cdot \vec{B} \equiv \frac{1}{\mathcal{V}} \sum_k \frac{-i\vec{k}}{|\vec{k}|^2} \cdot (\vec{B}(\vec{k}) \times \vec{B}^*(\vec{k})), \quad (3)$$

where $\mathcal{V}$ is the volume of space. Being a pseudoscalar and since we have not included CP violation, the expectation value of the helicity is zero. Nevertheless, to have an idea of the efficiency of this mechanism in generating non-trivial magnetic helicity we have computed its dispersion. Its time evolution is presented in Fig.3 where we also present the time dependence of the helicity dispersion in the Z-boson magnetic field. The latter has been rescaled by $\tan^2 \theta_W$ to make it agree with the electromagnetic

helicity for the initial configuration at $mt = 5$. While initially both helicities coincide, time evolution damps the Z-helicity while it enhances the Abelian magnetic one. This agrees with Vachaspati's picture, that predicted an initial correlation among both quantities, and subsequent decay of Z-strings, leaving remnant magnetic fields with non-trivial helicity. From our analysis we determine that the magnetic helicity dispersion is actually growing in time with a power-law behavior, $\sim t^{0.68(7)}$. A detailed study of the late time behavior of the system will be reported in Ref. [16]. We will discuss there the nature of the plasma during the first stages of reheating which shows evidence of electric charge separation. We will also present some evidence of the presence of inverse cascade in the magnetic power spectrum and the helicity dispersion.

In summary, in this paper we studied the preheating and early reheating periods of a model of hybrid inflation at the EW scale within the classical approximation. This is the first study of this sort including $SU(2)\times U(1)$. We have focused upon the generation of magnetic fields in this context and provided evidence that the mechanism proposed in 1991 by Vachaspati [4] is at work. Spatial gradients of the Higgs field, seeded by those already present in the random Gaussian initial condition, act as sources of the magnetic field. The latter has a non-trivial helicity dispersion, correlated initially with the presence of Z-strings, that is growing with time.

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